Nash Equilibrium: Illustrations

Refresher: Derivatives

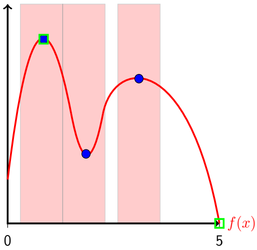
A derivative is the slope of the function at a given point.

- The Product Rule is:
  - $[f(x)g(x)]' = f'(x)\, g(x) + f(x)g'(x)$.
- The Quotient Rule is:
  - $\left[\frac{f(x)}{g(x)}\right]' = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$.
- The Chain Rule is:
  - $[f(g(x))]' = f'(g(x))\, g'(x)$.

$f(x)$	$f'(x)$
ax^b with $b \neq 0$	abx^{b-1}
ae^x	ae^x
$\ln(x)$	$\frac{1}{x}$

Extremum

- Consider the function $f(x)$ on the interval $[0, 5]$.
- An extremum is either a maximum or a minimum.



Definition

A **local extremum** of a function $f(x)$ over some domain D is a point that is an extremum in some neighborhood of $N \subseteq D$ of the domain D.

Definition

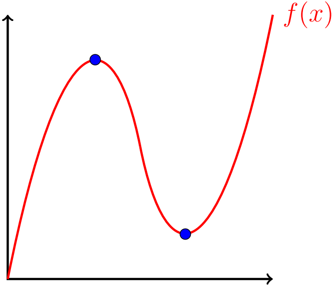
A **global extremum** of a function $f(x)$ over some domain D is a point that is an extremum over the entire domain D.

Finding Local Extrema

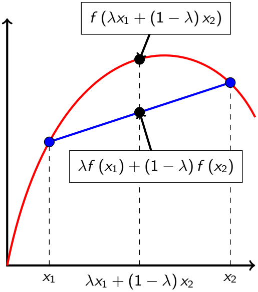
- At a local extremum,

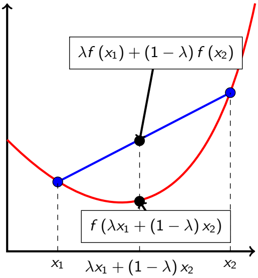
$$f'(x) = 0.$$

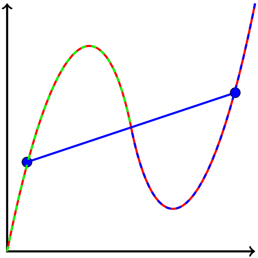
- This is called the first-order condition.
- The global maximum and minimum will always be at a local extremum or an endpoint.
- How one determines if there is a maximum or a minimum?



Concave and Convex Functions







Concave	Convex	Both
$f''(x) \leq 0$	$f''(x) \geq 0$	$f''(x)$ can be anything

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    - finally, compare that value to the value of the function at $x = \infty$ and $x = -\infty$.

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- To locate the maximum of a function on a constrained region $[x_1, x_2]$, find the unconstrained maximum, x_u^*.
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Exercise on Finding a Maximum

- Find the x that maximizes $-3x^2 + 6x - 4$ in the region $[-2, 0]$ (that is, $-2 \leq x \leq 0$).

Oligopoly

- **Oligopoly** refers to competition between a small number of sellers.
- Some examples are:

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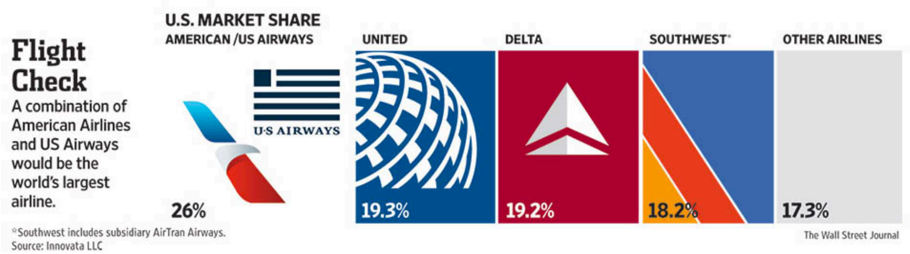

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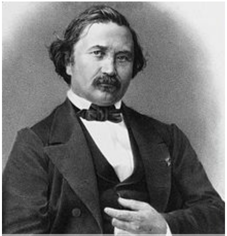
Oligopoly Models

- Economists did not use much math before the mid-19^{th} century.
- Classical economics studied extreme markets, such as:
  - perfectly competitive markets with an infinite number of buyers and sellers.
  - monopolies with an infinite number of buyers, but only one seller.
- Less extreme markets are more difficult to study because we need to take strategic considerations into account.
- Early duopoly models were precursors to the Nash equilibrium concept.


Cournot (1838)


Bertrand (1883)

Cournot Model of Oligopoly

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- For firm i to produce q_i units of the good, it will cost $C_i\left(q_i\right)$.
- Each unit of output is sold at a single market price, which depends on the market demand, which depends on the quantity produced by each firm; that is,

$$P\left(q_1, q_2, \ldots, q_n\right).$$

Cournot Model of Oligopoly (Cont.)

- The game consists of the following elements.
  - **Players**: There are $n \geq 2$ firms.
  - **Actions**: The levels of production are given by $q_i \in [0, \infty)$.
  - **Payoffs**: The profits for firm i are $\pi_i (q_i, q_{-i}) = q_i P (q_i, q_{-i}) - C_i (q_i)$.

Exercise on Cournot Duopoly

- Consider a duopoly.
- Firms face cost function $C_i\left(q_i\right) = cq_i$.
- Let the total market production be denoted by $Q = q_1 + q_2$.
- The inverse demand function is

$$P(Q) = \begin{cases} \alpha - Q & \text{if } Q \leq \alpha \\ 0 & \text{if } Q > \alpha. \end{cases}$$

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- Your tasks:

1. Write out the profit function for firm 1; that is, $\pi_1(q_1, q_2)$.
2. If firm 2 produces $q_2 = 0$, how much does firm 1 want to produce? What are their profits?
3. If firm 2 produces $q_2 > 0$, how much does firm 1 want to produce?
4. What is the Nash equilibrium of this game? What are their profits?

Cournot Model

Let $P(Q)$ be the inverse demand function; that is, the price as a function of the aggregate quantity Q.

For example, $P = 2 - Q$.

Let $c = 1$ be the unit cost to firm i of producing the good.

- Assume firms face constant marginal costs.

Cournot Model (Cont.)

Firm 1's profit function is:

$$\begin{aligned} \pi_1 &= \text{Revenue} - \text{Total Cost} \\ &= P(q_1 + q_2) \cdot q_1 - 1 \cdot q_1 \\ &= [2 - (q_1 + q_2)] \cdot q_1 - q_1 \end{aligned}$$

Firm 1's revenue is also changing with Firm 2's choice of output q_2.

Firm 2's profit function is similar; thus, it also depends on Firm 1's output level.

Nash Equilibrium

Use the **Nash equilibrium** concept to locate a profile of strategies that when holding the strategy of the other firm fixed, no firm can obtain a higher payoff (profit) by choosing a different strategy.

In the Cournot model, a firm strategy dictates choosing a quantity.

We need to find the profit maximizing quantity of one firm holding the other firm's choice of quantity fixed.

Cournot Model

For any given level of Firm 2's output level q_2^*, Firm 1 must choose an output level to maximize profits so that

$$\max_{q_1}[2-(q_1+q_2^*)]\cdot q_1 - q_1.$$

How does Firm 1 do it? As usual, by setting $MR = MC$.

$$MR = \frac{dTR}{dq_1} = \frac{d(2q_1 - q_1^2 - q_2^* q_1)}{dq_1} = 2 - 2q_1 - q_2^*$$

$$MC = \frac{dC}{dq_1} = \frac{dq_1}{dq_1} = 1$$

Cournot Model (Cont.)

So, we solve for Firm 1's profit maximizing q_1^* as a function of Firm 2's output level q_2^*. Thus,

$$MR = MC$$
$$2 - 2q_1^* - q_2^* = 1$$
$$q_1^* = \frac{1}{2}(1 - q_2^*)$$

This is called the **Cournot Best Response Function.**

Let us graph this ($q_2^* = 0 \rightarrow q_1^* = 1/2$ and $q_2^* = 1 \rightarrow q_1^* = 0$).

Cournot Model (Cont.)

Can you solve Firm 2's profit maximizing output q_2^* in terms of Firm 1's output level q_1^*?

Cournot Model (Cont.)

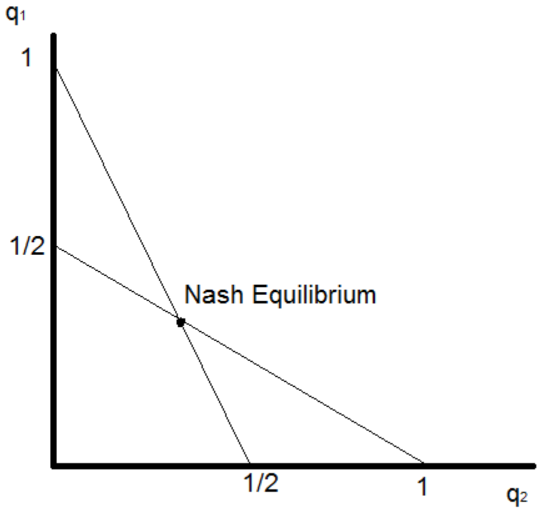
Can you solve Firm 2's profit maximizing output q_2^* in terms of Firm 1's output level q_1^*?

Firms choose a symmetric strategy so that $q_2^* = \frac{1}{2}(1 - q_1^*)$.

Graph the best response function of Firm 2 as well.

To find the Nash Equilibrium, we want the two strategies to "match."

Cournot Solution Graph



Cournot Solution

Now, we solve a system of two equations and two unknowns.

By substitution,

$$q_1^* = \tfrac{1}{2}(1 - q_2^*)$$
$$\Rightarrow q_1^* = \tfrac{1}{2}\left(1 - \tfrac{1}{2}(1 - q_1^*)\right)$$
$$\Rightarrow q_1^* = \tfrac{1}{2} - \tfrac{1}{4} + \tfrac{1}{4}q_1^*$$
$$\Rightarrow \tfrac{3}{4}q_1^* = \tfrac{1}{4} \Rightarrow q_1^* = \tfrac{1}{3}$$

Substituting it back, we find $q_2^* = (1 - 1/3)/2 = 1/3$.

So, both firms produce $1/3$.

Cournot Solution (Cont.)

So we have $q_1^* = q_2^* = \frac{1}{3}$.

Total quantity produced in the market: $Q = q_1^* + q_2^* = 2/3$.

Given the inverse demand function,

$P(Q) = 2 - Q = 2 - 2/3 = 4/3$.

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- Firms face cost function $C_i\left(q_i\right)$.

Bertrand Model of Oligopoly (Cont.)

The Game
Players
$n \geq 2$ firms
Actions
$p_i \in [0, \infty)$
Payoffs
$\pi_i = \begin{cases} q_m p_m - C_i(q_m) & p_i = p_m \\ 0 & \text{else} \end{cases}$
where $q_m = \frac{D(p_m)}{N_m}$

Exercise on Bertrand Duopoly

- Consider a duopoly.
- Firms face cost function $C_i\left(q_i\right) = cq_i$.
- The demand function at price P,

$$D(P) = \begin{cases} \alpha - P & \text{if } P \leq \alpha \\ 0 & \text{if } P > \alpha. \end{cases}$$

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- Your tasks:

1. Write out the profit function for firm 1; that is, $\pi_1\left(p_1, p_2\right)$.
2. If firm 2 sets $p_2 = \infty$, what price does firm 1 want to set? What are their profits?
3. If firm 2 sets a price of p_2, what is the best response for firm 1 (remember firm 2 can set any $p_2 \in [0, \infty)$).
4. What is the Nash equilibrium of this game?

Bertrand Competition

Suppose that in a duopoly setting, the inverse demand function is $P(Q) = 2 - Q$. Then, we can write the demand function as:

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In the Bertrand model, Firms 1 and 2 set prices p_1 and p_2.

We assume that each firm can satisfy all the demand for its product (i.e. no capacity constraints).

In addition, since products are homogenous, consumers purchase only from the firm with the lowest price.

- For example, if $p_1 < p_2$, then no one buys from Firm 2; that is, $q_2^* = 0$.
- Instead, everyone buys from Firm 1; that is, $q_1^* = 2 - p_1$.
- If $p_1 = p_2$, then we assume that the two firms are splitting the demand evenly.

Bertrand Competition (Cont.)

Let us assume that the cost of producing output is $c = 1$, which is common to both firms.

Firm $i = 1, 2$'s profit is:

$$\text{If } p_1 < p_2, \quad \pi_1 = TR - TC = (p_1 - c)q_1^*, \pi_2 = 0$$

$$\text{If } p_1 > p_2, \quad \pi_1 = 0, \pi_2 = TR - TC = (p_2 - c)q_2^*$$

$$\text{If } p_1 = p_2, \quad \pi_1 = \pi_2 = (p_i - c)Q^*/2$$

We need to figure out the equilibrium pricing strategy for each firm.

Bertrand Best Response

We have learned about Cournot best response functions.

We use the Nash equilibrium concept here as well. In other words, let us locate one firm's profit maximizing pricing strategy holding the other firm's pricing strategy fixed.

Let us analyze the Firm 1's best strategy, case by case, given that Firm 2's strategy is p_2.

Bertrand Best Response (Cont.)

If $p_2 > c$, what is the firm's best pricing strategy?

- If $p_1^* > p_2$: no one buys from Firm 1, thus $\pi_1 = 0$.
- If $p_1^* < c$: everyone buys from Firm 1, thus $\pi_1 = (p_1^* - c)D(p_1^*) < 0$.
- If $p_1^* = p_2$: demand is split evenly, thus $\pi_1 = (p_1^* - c)D(p_1^*)/2$.
- If $c < p_1^* < p_2$: everyone buys from Firm 1. p_1^* can be just tiny bit lower than p_2, which implies that Firm 1 gets the entire market.

Firm 1's best strategy is always to undercut Firm 2 by lowering the price just a tiny bit.

Bertrand Best Response (Cont.)

If $p_2 < c$, what is the firm's best pricing strategy?

- If $p_1^* > p_2$: no one buys from Firm 1, thus $\pi_1 = 0$.
- If $p_1^* < p_2$: everyone buys from Firm 1, thus $\pi_1 = (p_1^* - c)D(p_1^*) < 0$.

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Firm 1's best strategy is to set $p_1^* \geq c > p_2$ to get zero profit. Better zero (economic) profit than incurring a loss.

If $p_2 = c$, what is the firm's best pricing strategy?

- If $p_1^* > p_2 = c$: no one buys from Firm 1, thus $\pi_1 = 0$.
- If $p_1^* < p_2 = c$: everyone buys from Firm 1, thus $\pi_1 = (p_1^* - c)D(p_1^*) < 0$.
- If $p_1^* = p_2 = c$: demand is split evenly, thus $\pi_1 = (p_1^* - c)D(p_1^*)/2 = 0$.

Bertrand Best Response (Cont.)

Firm 2's best response is the same, holding Firm 1's price fixed. What inference can we then make?

- No firm would choose $p < c$, because it's always better to earn nothing than incur a loss.
- If one firm has $p > c$, the other firm can always undercut.
- When does undercutting stop? When $p_1^* = p_2^* = c$. Is this a Nash equilibrium?
- Given $p_1^* = c$, can Firm 2 do any better? Given $p_2^* = c$, can Firm 1 do any better?

In this particular case, the Bertrand outcome is $p_1 = p_2 = 1$, $q_1 = q_2 = 1/2$, so $Q = 1$.

Cournot vs. Bertrand Models

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- Betrand Model
    - In an industry where quantity is easy to adjust, then, Bertrand may be a better model. For instance, in smartphone applications.

Hotelling Model

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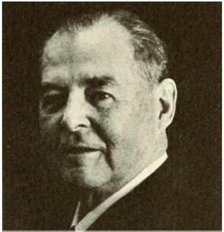
- Hotelling proposed a model of spatial competition.



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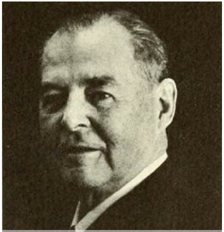

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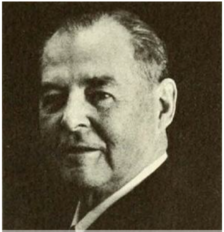

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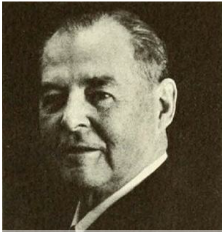

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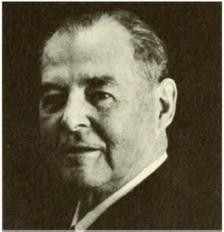
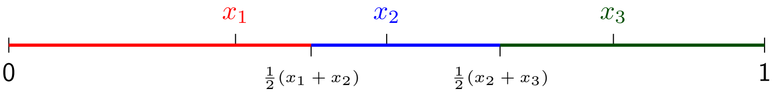


Hotelling (1929)

Hotelling Model (Cont.)

- **The Game:** (framed as electoral competition)
  - **Players:** n candidates.
  - **Actions:** location $x_i \in [0, 1]$.
  - **Payoffs:**
    - People vote for the closest candidate.
    - Win $\rightarrow n$.
    - Tie with k other candidates $\rightarrow n - k$.
    - Loss $\rightarrow 0$



Exercise on Hotelling Model

- There are two candidates.
- There is a continuum of citizens spread evenly on $[0, 1]$.
- Choose location $x_i = [0, 1]$.
- Payoffs (only care about the outcome):
  - 2 for a win,
  - 1 for a tie,
  - and 0 for a loss.

Exercise on Hotelling Model

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- Your tasks:
  1. If candidate 2 chooses $x_2 = 1$, what is candidate 1's best response?
  2. If candidate 2 chooses $x_2 = 0.75$, what is candidate 1's best response?
  3. Write out the best response for player 1.
  4. What is the Nash equilibrium of this game?
  5. What if $1/4$ of the citizens are located at 0 and the other $3/4$ are spread evenly across $[0, 1]$. What is the Nash equilibrium?

Hotelling's Location Model

Consumers view each firm's product as having a particular location in geographic or product space.

Hotelling developed a model to explain the location and pricing behavior of firms.

Let us model a town as a straight line (running East to West), and let us suppose that consumers' homes and firms (like gas stations) are located along the line.

Consumers are all the same, homes are evenly distributed, lining up east to west. Firms choose their location on the line. Assume the following.

- There are 2 firms (A and B) that sell the same product.
- It is costly for consumers to travel (time, gas).
- Consumers choose to buy from the nearest firm.

Location Model: Consumers

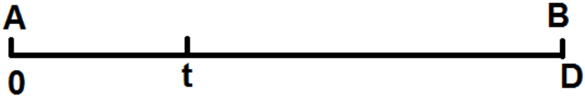
Suppose the travel cost for a consumer is $c > 0$ per mile distance traveled.

<u>Question</u>

Suppose Firm A & Firm B are located on the opposite ends of the town. Furthermore, the firms charge the same price.

Which households does each firm capture?

Let's take a look at one consumer who is located at the t mile-mark on the road. Which store would he choose?



Travel costs

- To store A :

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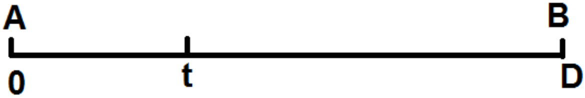

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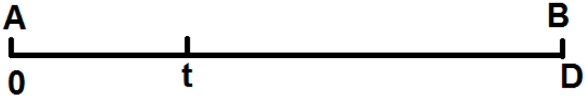
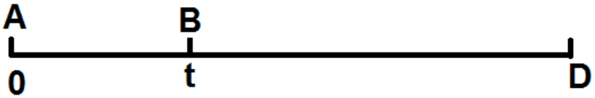


Travel costs

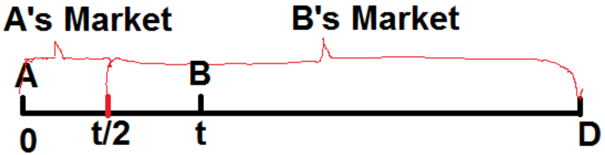
- To store A : tc
- To store B : $(D - t)c$

Location Model: Consumers (Cont.)

What if Firm A & Firm B are located as in the graph below?



Which households does each firm capture?



So Firm B captures more market by locating closer to A.

Location Model: Firm Location

Firms can make one of two decisions: change location or charge a different price.

Assume that we fix the price of the good first. Let us take a look at the location choice.

- Given B's location, where would A locate?
- How would B respond?

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- Locate as close to the other firm as possible.
- Just a tiny bit closer to the side with more consumers.
- What is the Nash equilibrium?

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- Given B's location, where would A locate?
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- Locate as close to the other firm as possible.
- Just a tiny bit closer to the side with more consumers.
- What is the Nash equilibrium?
- For both firms to locate in the middle of the line. Check whether either of the two firms has an incentive to change its location.
- What do locations of real gas stations look like?

Location Model: Price

Second, let us fix firms' locations. Firm A is at Location 0 and Firm B is at Location D. Allow the firms to choose different prices p_A and p_B.

We need to know consumer's utility to determine who they would buy from.

A consumer has some value attached to the good he buys. Let us say this value is denoted by V and that the consumer is located at mile-mark t.

- Consumer's utility if he buys from Firm A is $V - ct - p_A$.
- Consumer's utility if he buys from Firm B is $V - c(D - t) - p_B$.
- Consumer's utility if he does not buy is 0.

Notice that if prices are too high, consumers will choose not to buy!

Indifferent Consumer

For any given p_A and p_B, we need to know how many consumers each firm captures.

Thus, we find the indifferent consumer; that is, the consumer who is indifferent between going to Firm A or to Firm B.

- Everyone to the Left of the indifferent consumer, goes to Firm A.
- Everyone to the Right of the indifferent consumer, goes to Firm B.

How do we find the indifferent consumer's location?

Indifferent Consumer (Cont.)

The consumer is at location t^*, where

$$V - ct^* - p_A = V - c(D - t^*) - p_B$$
$$-ct^* = V - c(D - t^*) - p_B - V + p_A$$
$$-ct^* = -cD + ct^* - p_B + p_A$$
$$2ct^* = cD + p_B - p_A$$
$$t^* = \tfrac{cD + p_B - p_A}{2c} = \tfrac{D}{2} + \tfrac{p_B - p_A}{2c}.$$

So the location of the indifferent consumer depends on D, c, p_B and p_A.

Firm A gets all the consumers from 0 to t^*, and Firm B gets all the consumers from t^* to D.

Firm's Profits

Suppose firms have a product cost of $\$0$ per unit of good sold, and suppose every consumer buys 1 and only 1 product each time. In addition, the consumers are uniformly distributed over the line.

Firm A's profits are

$$\pi_A = p_A t^* = p_A \left(\frac{D}{2} + \frac{p_B - p_A}{2c} \right).$$

Firm B's profits are

$$\pi_B = p_B(D - t^*) = p_B \left(\frac{D}{2} - \frac{p_B - p_A}{2c} \right).$$

How can we derive each firm's best response function?

Firm's Best Response Function

Holding p_B fixed, we need to calculate Firm A's best response in choosing p_A to maximize π_A. Thus, we set $MR = MC$; since $MC = 0$, then $MR = 0$.

$$\pi_A = \frac{D}{2} p_A + \frac{p_B p_A}{2c} - \frac{p_A^2}{2c}$$

$$\frac{d\pi_A}{dp_A} = 0 = \frac{D}{2} + \frac{p_B}{2c} - \frac{p_A}{c}$$

$$0 = cD + p_B - 2p_A$$

$$p_A = \frac{cD + p_B}{2}.$$

Similarly, Firm B's best response is $p_B = (cD + p_A)/2$.

So, a firm's optimal price is increasing with consumer's travel cost c, the distance between two firms D, and the rival store's price.

Nash Equilibrium

Find the prices where the two best responses match!

$p_A = \frac{cD+p_B}{2}$

$p_B = \frac{cD+p_A}{2}$

By substitution:

$$p_A^* = p_B^* = cD.$$

So in equilibrium both firms set the same price.

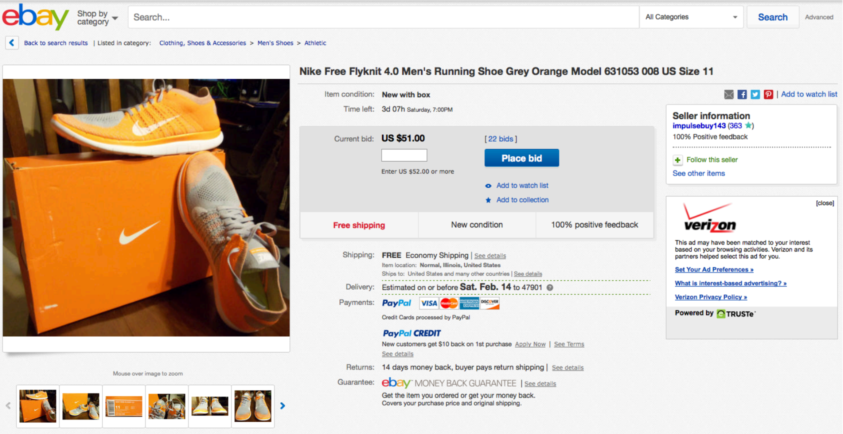
- The higher the consumer's travel cost c, the higher the prices.
- The bigger distance between the two firms D, the higher the prices.

Location Model: Price and Location

- Firms locate near each other, thus compete on price.
- The further away firms are from each other, the higher the prices that they are able to set; recall, prices increase with D.
- If $D = 0$, firms are on top of each other, setting prices at marginal cost which leads to zero profits.
- If D is large, they are sufficiently far, which implies that they act as local monopolies.
- Firms would like to locate as far away from each other as possible.

Auctions

- **Auctions** are a public sale in which goods or property are sold to the highest bidder.
- There are different types of auctions.
  - In first-price auction, the highest bidder wins and pays the bid.
  - In second-price auction, the highest bidder wins and pays the second highest bid.
  - In an all-pay auction, the highest bidder wins and everyone pays their bid.
  - In a Dutch auction, the auctioneer starts with a high price that gradually decreases until someone accepts and pays that bid.
  - In an English auction, the auctioneer starts with a low price that gradually increase until only one person is left who pays that bid.



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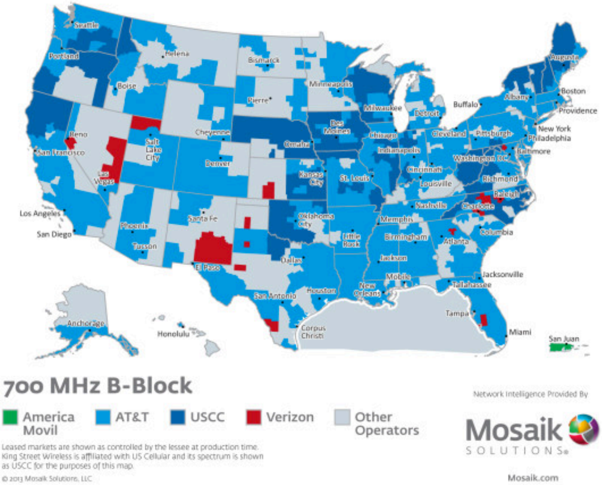
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Auction Game

- **Players** -

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    - Everyone knows **all** values.
- **Actions** - A player can bid $b_i \in [0, \infty)$.
- **Payoffs** - They depend on the auction type.

Auction Game (Cont.)

First-Price Payoffs

$$u_i\left(b_i, b_{-i}\right) = \begin{cases} v_i - b_i & \text{if } b_i > b_j \text{ for all } j \neq i \\ 0 & \text{otherwise.} \end{cases}$$

Second-Price Payoffs

$$u_i\left(b_i, b_{-i}\right) = \begin{cases} v_i - w & \text{if } b_i > b_j \text{ for all } j \neq i \\ 0 & \text{otherwise.} \end{cases}$$

(where w is the second highest bid)